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Combining Discovery and Direct Instruction Strategies in Computer-Based Teaching of Mathematical Problem Solving

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Discovery and direct instruction techniques were combined in a computer-based lesson that taught the relationship between parabolic graphs and their equations. A 2 x 2 factorial study design was applied to 152 university students enrolled in a basic Algebra class. The effects of two direct instruction techniques were determined: the cognitive routine and the Corrective Feedback Paradigm. The findings indicated that combining the two strategies in one lesson is instructionally powerful and effective. The effects of both the cognitive routine (p = .004) and the Corrective Feedback Paradigm (p = .005) were significant.

Direct instruction (Engelmann & Carnine, 1982) begins with an assumption that the environment is the primary variable that accounts for what a learner learns. Typically, when students fail to learn, the focus of attention is on them — their motivation, study habits, abilities, disabilities, etc. Direct instruction theory focuses attention on the instruction, or *communication,* students receive.

For example, in introducing a new concept to a student, did the instruction present a selection of positive examples that adequately demonstrated the range of the concept? Did the instruction present a selection of *negative* examples that adequately demonstrated the boundary of the concept? Was the instruction efficient?

A direct instruction analysis of communications seeks principles for the logical design of teaching sequences that efficiently transmit knowledge. Such teaching sequences also minimize the learning of misrules, over-generalizations, and under-generalizations.

Direct instruction theory provides a method of analyzing cognitive knowledge to determine effective communication rules/formulas for presenting that knowledge. A direct instruction analysis of cognitive knowledge results in a taxonomy of knowledge structures ranging from basic concepts (such as the meaning of the word "red," or the word "under") to complex forms of knowledge (such as the cognitive routine in this study). Each knowledge structure is associated with communication rules/formulas that effectively communicate that knowledge.

The cognitive routine, a direct instruction strategy, is used to present instruction in complex problem solving (in this study, the determination of equations for parabolic graphs). A cognitive routine is a set of steps, or algorithm, that leads to the solution of a problem. A cognitive routine *supplies* the student with a reliable method of deriving correct answers.

To teach a cognitive routine, tasks are designed that make all necessary steps leading to the desired outcome overt. The learner must make an overt response on each step of the routine. Because of this, if the learner makes a mistake, we know exactly what it is, and can provide appropriate remedial instruction. In this study the cognitive routine provides the student with a method for determining equations of parabolic graphs.

Increasing ratio review provides extra practice on missed items in a drill. When a student makes an incorrect response, the item reappears in the drill according to a specified review schedule. If the schedule were a 1-later, 3-later, 5-later review schedule, the missed item appears again immediately after it is missed. If it is answered correctly, it appears again three items later. If answered correctly again, it appears five items later. The missed item is also placed at the end of the list to satisfy the retirement criterion. The instructional designer specifies the number of times the student must answer the item correctly, without intervening mistakes, before it is retired from the list.

The Corrective Feedback Paradigm (Siegel & Misselt, 1984) is a computer-based drill strategy based on direct instruction strategies and instructional feedback methods (Kulhavy, 1977). In this study, CFP implements increasing

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ratio review and the retirement criterion previously described.

Intrinsic feedback is a component of intrinsic models, described by Dugdale (1982, 1983) and Dugdale and Kibbey (1977, 1980). In intrinsic models, environments are created that encourage students to explore innovative problem solving strategies. Students can manipulate and explore the environment in any way they choose, and are guided by intrinsic feedback. Intrinsic feedback is characteristically direct, relevant and diagnostic. It can provide maximum visual feedback to students about their responses to problem solving tasks. In this study, students can compare the graphs of their equations to the target graphs (see Figure 1).

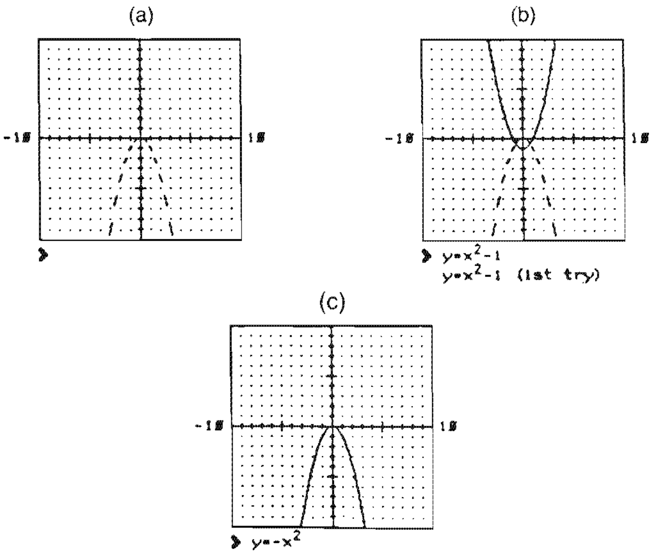

Figure 1. Intrinsic feedback in deriving equations for parabolas: (a) the target graph, (b) an incorrect response, and (c) a correct response.

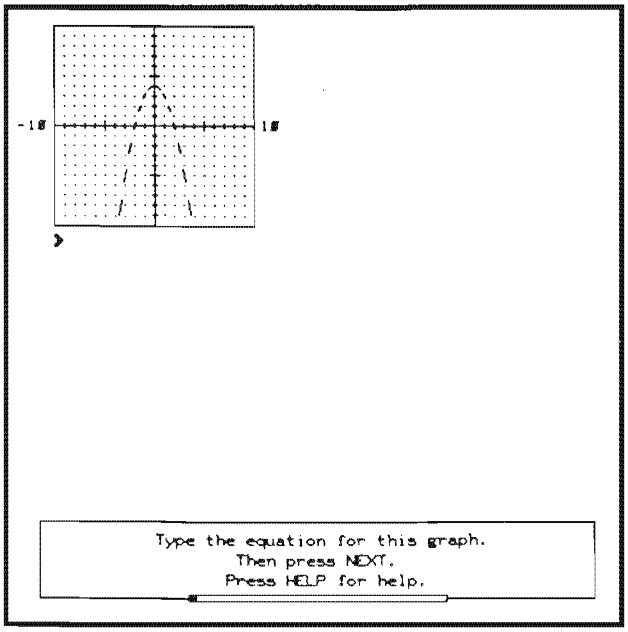

Figure 2. Presentation of a target graph.

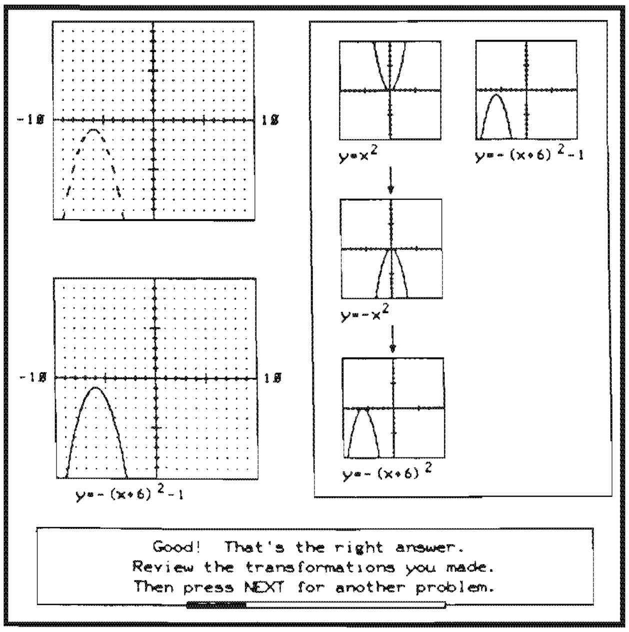

Figure 3. A completed cognitive routine. Target graphs are presented at the lower left. Graphs at the right record students' step-by-step progress.

Method

Lesson Design

In the lesson, the student is shown a parabolic graph, and asked to type the equation for the graph (see Figure 2). The student must understand how the different parts of the equation cause a change in both the position and orientation of the graph (graphs can be off the axes, inverted, turned on their sides, etc.). The student needs to understand four basic concepts in order to perform successfully on the tasks. They are:

I. changing the axis of symmetry ("x=" or "y=").
II. inverting the parabola ($y = -x^2$).
III. moving the graph along the axis which *is not* the axis of symmetry — the most difficult, counter-intuitive concept of the four ($y = (x - b)^2$).
IV. moving the graph along the axis of symmetry ($y = x^2 + c$).

A "y=" graph will be oriented up or down, while an "x=" graph will be oriented sideways. A minus sign in front of the squared term will invert a graph. The last two concepts involve moving the graph up or down or laterally relative to the axes. The third concept is difficult, because the graph moves in the *opposite* direction of the sign of "b." Concepts II, III, and IV apply to "x=" graphs as well as "y=". When students type an incorrect equation twice or request help, they are led step by step through the necessary transformations of the cognitive routine and must produce the correct response for each step (see Figure 3). The cognitive routine is as follows:

- Begin with "$y = x^{2}$" or "$x = y^2$," depending on whether the graph is oriented up/down or sideways.
- Transform the equation by applying concepts II, III, and IV, *in the order listed.*
- Not all graphs incorporate all transformations—any unnecessary transformations are simply *dropped* from the routine.

If students type an incorrect answer twice while being led through the cognitive routine, they are given the answer and required to type it. Completion of the cognitive routine results in the correct equation for the original task and a graphic map of how it was achieved (see Figure 3).

The order of tasks presented in the lesson appears in Table 1. Each task represents a set of parabolas (or generalization) that is defined by the general equation (except as noted). Each generalized task involves one or more of the above described concepts.

The Corrective Feedback Paradigm used in this lesson incorporates a 1-3-5 later increasing ratio review schedule. Items are selected from the appropriate generalized class for each item that represents a generalization. Items that are non-generalized tasks or specific instances of a generalization, are repeated as single-item tasks. The retirement criterion is set to ensure that all students continue with the drill until they have spent 40 minutes in the lesson.

Table 1. Lesson Tasks and Component Concepts.

Task Equation	Concepts
1. $y = x^2$ (n)	I
2. $y = -x^2$ (n)	I, II
3. $y = x^2 + c$, $c \neq 0$	I, IV
4. $y = -x^2 + c$, $c \neq 0$	I, II, IV
5. $y = (x - b)^2$, $b \neq 0$	I, III
6. $y = (x - b)^2 + c$, $b \neq 0$, $c \neq 0$	I, III, IV
7. $y = -(x - b)^2$, $b \neq 0$	I, II, III
8. $y = -(x - b)^2 + c$, $b \neq 0$, $c \neq 0$	I, II, III, IV
9. $x = y^2$ (n)	I
10. $x = -y^2$ (n)	I, II
11. $x = y^2 + c$, $c \neq 0$	I, IV
12. $x = -y^2 + c$, $c \neq 0$	I, II, IV
13. $x = (y - b)^2$, $b \neq 0$	I, III
14. $x = (y - b)^2 + c$, $b \neq 0$, $c \neq 0$	I, III, IV
15. $x = -(y - b)^2$, $b \neq 0$	I, II, III
16. $x = -(y - b)^2 + c$, $b \neq 0$, $c \neq 0$	I, II, III, IV

NOTE. (n) denotes a non-generalized task. Concepts are explained in the text.

Intrinsic feedback is used in this lesson because it indicates whether or not an answer is correct as well as supplies a maximum of visual information regarding the nature of any errors (see Figure 1). Students can visually compare their graph with the target graph. The entire intrinsic model is not used, however. When a student errs twice in responding to a particular task, he is immediately led through the cognitive routine. A purely intrinsic model would allow the student to experiment and explore.

Research Design

A 2 x 2 factorial design is used in this study. The two factors are the cognitive routine strategy and the Corrective Feedback Paradigm. Each factor has two levels represented by the presence or absence of the instructional technique (CR and No CR, CFP and No CFP). The task order in Table 1 is the order of presentation used in all four treatment groups.

In each of the groups, the student was allowed two tries on each task. In the CR groups, after two consecutive misses, the student was led through the cognitive routine to the correct equation. In the CFP groups, after two consecutive misses, the student was supplied with the correct answer, required to type it, and received 1-3-5 increasing ratio review on the item. In the CR/CFP group, after two consecutive misses, the student was led through the cognitive routine for the item and received increasing ratio review. In the No CR/No CFP group, after two consecutive misses, the student was supplied with the answer and required to type it.

The CR/CFP group combined an overt cognitive routine with a computer-based drill strategy (CFP). The cognitive routine was used only when required (the student missed an item twice or pressed HELP). The cognitive routine required the student to respond overtly to each step of the routine. Engelmann and Carnine state that any difficult components of a cognitive routine should be allotted more practice time (p. 214). CFP provided for this automatically (increasing ratio review) and individually (each student received only the necessary review). The CR/CFP group represented a potentially optimal combination of instructional techniques.

Subjects

The experimental subjects were 152 students enrolled in one of six sections of a university basic Algebra class. The course material covered prior to the study involved preparatory study of quadratic equations and graphing. All subjects were at the same place in the course material when they participated in the study.

Table 4. Analysis of Variance - Multiple-Choice Test Scores.

Source of Variation	SS	df	MS	F	p
Within Cells	591.47	148	4.00		
Constant	7170.63	1	7170.63	1794.25	0
CR	.42	1	.42	.11	.746
CFP	17.79	1	17.79	4.45	.037
CR by CFP	1.68	1	1.68	.42	.517

NOTE. Analysis of variance of multiple-choice test scores with CR and CFP as factors.

(CR) was not significant ($p = .746$). The Corrective Feedback Paradigm was significant ($p = .037$). No interaction effect was demonstrated ($p = .517$). The pretest as covariate did not have a significant effect in predicting the multiple-choice test scores ($p = .629$). Therefore, an analysis of covariance was not indicated.

Discussion

The data analysis in this study indicates that both the cognitive routine and the Corrective Feedback Paradigm had significant effects on written posttest scores, while only CFP had an effect on the multiple-choice test scores. This could be due to the different type and timing of the two tests. The written posttest, given the day after the lesson, required a production response. It is possible that the cognitive routine helped the student to remember the concepts involved and to produce the required equations. Memory was probably not as much of a factor on the multiple-choice test given immediately after the lesson. This interpretation suggests that CR and CFP should be used where longer retention and production responses are required.

The difference in the regression slopes for the No CR and CR groups suggests further research. The data analysis indicates pretest performance was not predictive of posttest performance for students who received the cognitive routine. It is possible that students of varying ability levels can perform equally well in learning and applying a clear solution algorithm to a well-defined set of problems.

The importance of the techniques that were *not* experimentally manipulated in this study require mention. Intrinsic feedback gave students immediate and powerful visual information about their attempts to solve problems. The careful sequencing and introduction of tasks provided a gradual and logical presentation of the concepts to be learned. When students pressed ANS, they were required *to remember* the answer before they could type it (the answer did not remain on the screen). Screen designs and instructional wording were made as efficient and unobtrusive as possible. While the strengths of the computer were exploited, the student was freed, as much as possible, from having to attend to the characteristics of the medium itself.

The design techniques implemented in this study provided effective instruction for well-structured solution algorithms. Future research could investigate the appropriateness of these techniques to other problem-solving tasks. In knowledge spaces where solution algorithms are less-structured (e.g., the social sciences), these techniques could lose effectiveness. Future research could establish the appropriateness of these techniques to other tasks and domains.

References

Dugdale, S. (1982, March). Green globs: A microcomputer application for graphing of equations. *Mathematics Teacher,* pp. 208-214.

Dugdale, S. (1983, March). There's a green glob in your classroom. *Classroom Computer News,* pp. 40-43.

Dugdale, S., & Kibbey, D. (1977). *Elementary mathematics with PLATO* (2nd ed.). Urbana, IL: University of Illinois, Computer-based Education Research Laboratory.

Dugdale, S., & Kibbey, D. (1980). *The fractions curriculum — PLATO elementary school mathematics project* (CERL Report E-17). Urbana, IL: University of Illinois, Computer-based Education Research Laboratory.

Engelmann, S., & Carnine, D. (1982). *Theory of instruction: Principles and applications.* New York: Irvington.

Hull, C., & Nie, N. (1981). *SPSS update 7-9.* New York: McGraw-Hill Book Company.

Kulhavy, R. C. (1977). Feedback in written instruction. *Review of Educational Research,* **47**(2), 211-232.

Siegel, M. A., & Misselt, A. L. (1984). Adaptive feedback and review paradigm for computer-based drills. *Journal of Educational Psychology,* **76**(2), 310-317.

Smith, S. G., & Sherwood, B. A. (1976). Educational uses of the PLATO computer system. *Science,* **192**, 344-352.